Resonant Frequencies and Harmonics

Resonance

If you've ever pushed anyone on a swing, you may have noticed that swings with short chains swing more quickly (higher frequency) while swings with longer chains swing more slowly (lower frequency). Each swing has what we like to call a **natural frequency**, or **harmonic frequency** — the frequency that it swings at automatically, of its own accord due to its design.

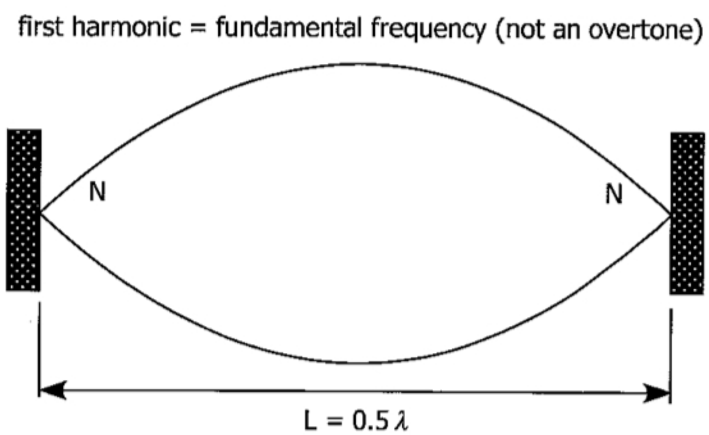
A **natural** (or **harmonic**) frequency is the lowest frequency at which an object, like a simple pendulum or a mass on a spring, will vibrate when it is allowed to vibrate freely.

If you were to push the person on the swing with a timing (frequency) that matches the natural frequency, you would observe a phenomenon called **resonance**. The size of the swings (which we would call amplitude) will get larger and larger with each push.

Resonance is the tendency of a system to oscillate at larger amplitudes at some frequencies than at others. These are known as the **resonant frequencies** of the system. At these frequencies, even small periodic driving forces can produce large amplitude oscillations.

Resonance and Standing Waves

The lowest frequency that will produce a standing wave results in a standing wave consisting of one complete loop, as pictured below.



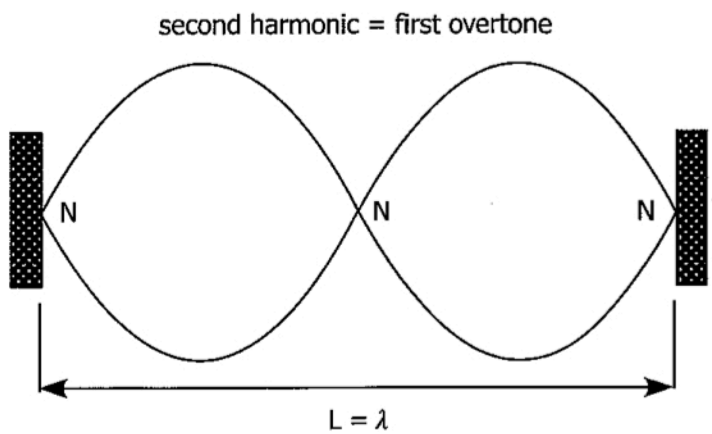
If you continue to vibrate the source at this frequency, the amplitude of the standing wave will grow larger and larger. This happens because standing waves occur at the resonant frequencies of the spring.

The lowest frequency that produces a standing wave is called the **fundamental frequency** (also known as the first harmonic frequency, or simply **first harmonic**). The fundamental frequency is the lowest natural frequency with which the object will vibrate.

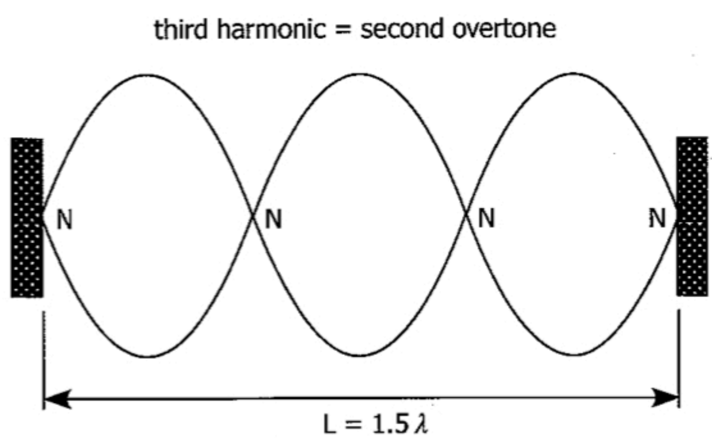
Overtones

Additional standing wave patterns will occur at frequencies that are whole number multiples of the fundamental frequency. These additional resonant frequencies are called **overtones**.

The first overtone occurs at a frequency that is exactly double the fundamental frequency (known as the **second harmonic**). At this frequency, a standing wave with two complete loops is produced, as shown below.



At a frequency that is three times the fundamental (known as the **third harmonic**), the second overtone is produced. At this frequency, a standing wave with three complete loops is produced, as shown below.



Example 1
A certain spring has a fundamental (first harmonic) frequency of $200\ Hz$. Complete the following table:

Frequency	**Number of Loops**	**Harmonic**	**Overtone**
$f_0 = 200\ Hz$	1	first	
$2f_0 = 400\ Hz$	2	second	first
$3f_0 = 600\ Hz$	3	third	second

Example 2
A guitar string has a length of $0.90\ m$ and a fundamental frequency of $250\ Hz$. The string is vibrating at the second harmonic.

a) What is the frequency of this vibration?

b) What is the wavelength of this wave?

c) What is the speed of this wave on the string?

Waves Worksheet #4

1. If the length of a string is doubled, by what relative amount has the fundamental frequency changed?

2. A piano string is $1.10\ m$ long. The fundamental frequency of the string is $131\ Hz$.
   a) What is the wavelength of the fundamental frequency?
   b) What are the frequencies of the first two overtones?

3. A string of length $2.50\ m$ is fixed at both ends. The speed of the wave is $85.0\ m/s$. What is the fundamental frequency of the wave?